An Error Rate Model of Relay Communications with Lossy Forwarding and Joint Decoding

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Abstract—This paper presents a link quality model for a wireless communication system with distributed turbo coding and lossy forwarding. The model maps the signal-to-interference-plus-noise ratio (SINR) of packet copies received from different links to a mutual information parameter and, in a second stage, converts it to a block error rate. We present the design and foundation of the model and validate its accuracy for different modulation and coding schemes over additive white Gaussian noise (AWGN) channels. The model accurately predicts the link-level performance at a low computational complexity and can be therefore used as a physical layer (PHY) abstraction for the computationally-intensive, simulation-based performance evaluation of various functionalities at higher protocol layers or at the system level. In order to illustrate the usage of the proposed model, we show the integration of the model into a protocol level simulator assessing the performance improvements of lossy forwarding and joint decoding.

I. INTRODUCTION

Cooperative communication with distributed turbo coding (DTC) has attracted great attention in the wireless communications research community. The conventional approach to DTC uses decode-and-forward (DF) relaying where the intermediate nodes decode and regenerate the signal transmitted from the source, such that the noise propagation can be avoided. However, it has been suggested in [1] and investigated in [2, and the references therein] that even if the data is incorrectly decoded at the relay, the forwarded estimates are still helpful for the reconstruction of the source information at the destination. Practical and efficient codes for DTC with lossy forwarding (LF) and joint decoding (JD) have been designed [3] and described in [4], [5].

Although the advantages of DTC with LF at the physical layer (PHY) are evident, message transfer protocols need to be re-designed in order to maximize the performance from the network perspective. This re-design includes various higher layer functionalities, such as multirate control, transmit power control, automatic repeat request (ARQ), and medium access control (MAC), as well as multipath routing protocols. Their design and comprehensive evaluation requires protocol and system-level simulations under a wide range of operating conditions and in various deployment scenarios. To evaluate the performance at higher protocol layers and system level, individual links need to be included into the simulation, but their detailed modeling is often computationally prohibitive. An abstraction of the PHY reduces the complexity of the overall simulation and accurately predicts the link-level performance.

The use of PHY abstractions for link performance prediction can be regarded as a common and well-accepted method for protocol- and system-level performance evaluation [6]. Conventional and simpler approaches typically assume a geometric signal-to-interference-plus-noise ratio (SINR) distribution over the network topology in order to predict an average link-level performance, e.g., for network planning. In comparison, PHY abstraction enables the accurate modeling of instantaneous channel and interference conditions at reasonable computational costs.

The main principle of the PHY abstraction is to map a channel quality measure to an error rate of a (link layer) packet by means of a look-up table. In wireless multicarrier systems, the abstraction includes the reduction of the quality measure values for the individual subcarriers to a single scalar. In this context, various mapping functions have been proposed (see [6] for an overview) and most of them follow one of the two approaches: Exponential Effective SINR Mapping (EESM) and the Mutual Information (MI) based link quality model.

In EESM a set of subcarrier SINRs are mapped to a single corresponding effective SINR [7]. However, EESM requires a fine-tuned adjustment factor for different modulation orders and code rates to reach high accuracy. The effective SINR can be approximated to a SINR for a flat-fading channel for which the Block Error Rate (BLER) can be determined. To overcome the need for a fine-tuned adjustment factor, [8] and [9] proposed a MI-based link quality model, which contains separate, subsequently applied models for modulation and coding. The modulation model maps the expected value of subcarrier SINRs to the symbol information (SI) in the MI domain [8], assuming additive white Gaussian noise (AWGN) as the channel model. In the coding model, the SI is collected to the received coded bit information (RBI) and then the BLER is determined by means of a RBI look-up table. It has been shown that the MI-based link quality model not only eliminates the dependency on the modulation scheme but also achieves a high accuracy for the BLER [10].

So far, the existing schemes consider a single link only. The error rate model proposed in this paper adopts the main principle of the MI-based PHY abstraction. While the scheme has originally been proposed for multicarrier transmission, we apply it to the case of multipath relay communication with LF and JD. Our model is still compatible to the generic link performance model [10] with its three stages of extraction of quality measures, compression of quality measures to a scalar...
The joint decoding strategy utilizes correlation knowledge among the relays’ data through global iteration [14].

A block diagram of a parallel lossy DF relaying model is depicted in Fig. 1, where $u_k = u \oplus e_k$ denotes the binary sequence at a relay $k$ after decoding. $\oplus$ indicates modulo 2 addition, $u$ is the original binary independent identically distributed (i.i.d.) information sequence from the source, and $e_k = [e_{k,1}, e_{k,2}, \ldots, e_{k,L}]$ is a bit flipping sequence defined as

$$e_{k,n} = \begin{cases} 1, & \text{with probability } p_k, \\ 0, & \text{with probability } 1 - p_k, \quad n = 1, 2, \ldots, L. \end{cases}$$

Each correlated version of the source sequence $u_k$ is interleaved and encoded by a twofold serially concatenated code (SNRCC) and second, a doped accumulator (ACC) [3], i.e., memory-1 systematic recursive convolutional code (SRCR) are deployed. The ACC is used to prevent an error floor at the relay memory-1 systematic recursive convolutional code (SRCC) are

The remainder of the paper is as follows: Sec. II describes the model of the parallel lossy DF system. Sec. III presents the design of the error rate model, which is validated for selected scenarios in Sec IV. Sec. V illustrates the application for system- and protocol-level simulations in a representative scenario. Sec. VI concludes the paper.

II. SYSTEM MODEL

A block diagram of a parallel lossy DF relaying model is depicted in Fig. 1, where $u_k = u \oplus e_k$ denotes the binary sequence at a relay $k$ after decoding. $\oplus$ indicates modulo 2 addition, $u$ is the original binary independent identically distributed (i.i.d.) information sequence from the source, and $e_k = [e_{k,1}, e_{k,2}, \ldots, e_{k,L}]$ is a bit flipping sequence defined as

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At the destination, soft demapping is applied by calculating the log-likelihood ratio (LLR) $L_{\hat{s}_k}$ with the received sequence $y_k$ and known channel state information. Each relay decoder in Fig. 2 has two matching BCJR algorithms [12]. In general, the joint decoder is structured in two main parts: (1) the local iteration (LI), where the relay sequence is decoded, and (2) the global iteration (GI), where the information exchange among all relay sequences is performed. In GI the LLR update function [3], based on the knowledge of $p_k$, updates the LLRs $L_{\hat{s}_k}$ accordingly. If the LLRs $L_{\hat{s}_k}$ are not improving anymore, the final estimation of $\hat{u}$ is determined by hard decision of the sum of all $L_{\hat{s}_k}$. For a detailed explanation of the joint decoder, the authors refer to [13].

III. ERROR RATE MODEL

The MI-based metric referred in Sec. I is well suited for the link-level abstraction of the system model where same data is received through multiple parallel links. Fig. 3 divides the abstraction model into two components, Blackbox #1 and #2, which illustrate the deterministic and stochastic parts, respectively. The idea is to use the MI metric given by Blackbox #1 to match with simulation based results stored in Blackbox #2. To be more specific, Blackbox #1 compresses a given set of parameters into a one single scalar, which is further mapped with simulation results in Blackbox #2. In the following we describe the content of both parts of the abstraction model.

A. Blackbox #1

Blackbox #1 was originally described in [4]. The BER after JD at the destination depends on the bit error probability (BEP) at the relay $k$ ($p_k$), the SINR of the link between relay $k$ and the destination ($SINR_{k_d}$), and the used modulation and coding scheme (MCS). Next, we show how these parameters are mapped to a single scalar $MI$.

A simplified version of the “achievable rate region” from [15] is shown in Fig. 4. The figure corresponds to a three-node system model, where the destination receives two copies of the same data packet, one directly from the source and another from a relay node. Here, the relay may be unable to decode correctly, and the resulting BER at the relay is $p$. The axes $R_1$ and $R_2$ represent the scaled channel.
constellation constrained capacities (CCCs) of the source-to-destination link and the relay-to-destination link, respectively, given by

\[ R_k = \frac{C(M_k, \text{SINR}_k)}{R_k} \]

with \( C(M_k, \text{SINR}_k) \) being the CCC of AWGN channel of the parallel link \( k \) depending on the modulation and \( \text{SINR}_k \). The scaling is performed with the spectral efficiency depending on the number of bits per symbol \( M_k \) in complex modulation and the channel code rate \( R_k \) used at the relay \( k \).

Theoretically, if the rate pair falls into the achievable rate region, error-free decoding at the destination is possible. Note that the rate region is information theoretic and independent of the code design and decoder implementation. We assume that the source sequences \( u_1 \) and \( u_2 \) have full entropy, i.e., \( H(u_1) = 1 \) and \( H(u_2) = 1 \), respectively.

![Achievable Rate Region](image)

**Fig. 4.** Rate region for a three-node system.

The bound of the admissible rate region consists of the straight line \( a \), i.e. \( R_1 + (1 - H_b(p_k)) = 1 \) and the curve \( b \). Here \( H_b(p_k) = -p_k \log_2 p_k - (1 - p_k) \log_2 (1 - p_k) \) denotes the binary entropy associated with probability \( p_k \). In order to simplify the model, we approximate the curve by the straight \( c \) as \( R_1 + (1 - H_b(p_k))R_2 = 1 \). By combining the two lines, we get an expression for the approximated admissible rate region as

\[ R_1 + \min\{1 - H_b(p_k), (1 - H_b(p_k))R_2\} \geq 1. \tag{2} \]

We further propose to generalize the combining of \( MI \) over any number of parallel incoming links so that

\[ MI = \sum_k M_k \]

\[ M_k = \begin{cases} R_k, & p_k = 0 \\ \min\{1 - H_b(p_k)R_k, 1 - H_b(p_k)\}, & p_k > 0. \end{cases} \tag{3} \]

In theory, when \( MI \) per bit is unity, there are no errors after JD. Note that if there are errors in the transmitter, corresponding \( MI \) remains always below unity. Otherwise, \( MI \) can get larger values. If all parallel links have suffered of errors before the last hop (a chief executive officer (CEO) problem [14]), an error floor occurs [16]. Blackbox \#1 also calculates the error floor \( p_{\text{floor}} \) by using the method described in [2, Sec. 4.1.4.1].

By employing Shannon’s source-channel separation theorem [17], the distortion (BER) at the output of the receiver can be lower bounded by the inverse of the binary entropy function [15] as

\[ p_e = H_b^{-1}(1 - MI), \tag{4} \]

which equals to zero when \( MI \geq 1 \). Now, the idealistic BEP \( p_{\text{ideal}} \) can be approximated as an upper bound of the error floor and \( MI \) as

\[ p_{\text{ideal}} = \max\{p_e, p_{\text{floor}}\}. \tag{5} \]

We call the mapping from the link-specific SINRs and transmitter BERs into the \( MI \) and \( p_{\text{ideal}} \) as Blackbox \#1. It employs a very simple deterministic mapping that is independent of the coding and decoding scheme, and requires no simulations. It predicts optimistic performance that corresponds to infinite packet sizes. It is also worth of noticing that due to the summation of MI’s in (3), the theory assumes that the erroneous bits do not interfere in decoding process, i.e., erroneous bits are treated as erasures.

### B. Blackbox \#2

It is well known that in block codes, the number of bit errors per packet is not uniformly distributed. In order to obtain MCS and packet length dependent BER performance results, we propose to concatenate Blackbox \#1 with Blackbox \#2 that employs a mapping obtained via simulations in the AWGN channel. Note that these results apply also in block fading channels. The idea is to obtain the random packet-by-packet decoding performance in terms of BER. In the following, we will describe the construction of mapping tables used in Blackbox \#2 for \( K \) parallel links.

1) **Construction of simulation tables for Blackbox \#2:** Since Blackbox \#1 is able to do the compression of channel quality measures to a single scalar, we can significantly reduce the number of simulation tables needed. For the simulations, we can perform the following simplifications: set \( p_k = 0 \) for all \( k \), since \( p_k \) is already taken into account when calculating \( MI \) through (3). Furthermore, since the simulation results and the theoretical results are compared only in terms of the sum of the MI’s over the links, we can set \( \text{SINR}_k = \hat{\gamma}, \forall k \), and sweep \( \hat{\gamma} \) through an appropriate SINR range. The simulations have to be performed separately for all MCSs considered.

Simulations were performed by sending 10,000 packets (8,000 bits each for QPSK, 8,400 for 16QAM and multi-rate) through the channel of each parallel link with orthogonal channel access. The output of the simulations was the number of errors in each packet after JD at the destination. Empirically, it was found that the errors are distributed approximately according to a distribution defined as

\[ P(\alpha, \mu, \sigma^2) = \alpha \delta(0) + (1 - \alpha)N(\mu, \sigma^2), \tag{6} \]

where \( \alpha \) is the probability that the number of errors in a packet is zero, \( \delta(\cdot) \) is the Dirac delta function, and \( N(\mu, \sigma^2) \) denotes the normal distribution with mean \( \mu \) and variance \( \sigma^2 \). An intuitive explanation for (6) is that \( \alpha \) portion of the packets are error-free, and for the remaining part, the error distribution follows Gaussian.

The parameter values \( p_k = 0 \) and \( \text{SINR}_k = \hat{\gamma} \) are fed to Blackbox \#1 which outputs \( MI \). For this given \( p_k \) and \( \text{SINR}_k \) values, we run simulations and then fit the simulation data to the distribution given above, resulting in numerical values
of parameters $\alpha$, $\mu$, and $\sigma^2$ corresponding MI output from Blackbox #1. As a result, we can construct a table with four columns that corresponds to $MI$, $\alpha$, $\mu$, and $\sigma^2$, respectively.

2) Generation of bit errors using Blackbox: Since the resolution for the MI values listed in the table is finite, linear interpolation can be used to obtain numerical values for the distribution parameters between the quantized points. For the cases where at least one error-free component exists at the relay, errors can be generated directly from (6). In the case of CEO problem, there exists an error floor which is further need to be considered by adding $E = E + Lp_{floor}$, where $L$ is the length of the transmitted sequence. The generation of the number of bit errors is described in Algorithm 1. The operation $E = \max\{0, E\}$ is used to prevent negative values of $E$.

Algorithm 1 Bit error generation for multi-path relay networks with LF and JD.

1: Input parameters $p_1, p_2, \ldots, p_K$, $\text{SINR}_1, \text{SINR}_2, \ldots, \text{SINR}_K$
2: Calculate $MI$ using (3)
3: Find $\alpha, \mu, \sigma^2$ matching the values on the tables with $MI$
4: Generate uniformly distributed random number $\rho \in [0, 1]$
5: if $\rho > \alpha$ then
6: Generate $E$ from $\mathcal{N}(\mu, \sigma^2)$
7: else
8: $E = 0$
9: end if
10: Calculate $p_{floor}$ using the method
   from [2, Sec. 4.1.4.1]
11: if $p_{floor} > 0$ then
12: $E = E + Lp_{floor}$
13: end if

IV. VALIDATION

In this section, we will analyze the accuracy of Blackbox in terms of BER and BLER in AWGN channel. Two different MCSs are used: quadrature phase shift keying (QPSK) with channel code rate 1/2 and 16-ary quadrature amplitude modulation (QAM) with channel code rate 3/4. The channel code parameters are shown in Table I. Natural mapping and modified set partitioning mapping (MSP) are used for QPSK and 16QAM, respectively.

We define four test cases:
- Case 1: QPSK with three parallel links: $p = [p_1, p_2, p_3]=[0 0 0]$.
- Case 2: Two parallel links, one uses QPSK and one uses 16QAM: $p=[0 0 1]$.
- Case 3: QPSK with three parallel links: $p=[0.01 0.02 0.05]$.
- Case 4: QPSK with three parallel links: $p=[0.01 0.02 0.05]$.

Cases 1 and 2 represent cases where exactly the same data is transmitted through independent links. From the relay network perspective, this means that all routes are error-free before the last hop (relay-destination link). Case 3 is a case where one of the routes is error-free and two of them have suffered from errors before the last hop. Case 4 represents a CEO problem.

The first step is to validate the error distribution defined in (6). The error distributions for Case 1 obtained using Blackbox and chain simulations with three SNR values are shown in Fig. 5. The SNR values are chosen from the waterfall region, since it is the most difficult region to track. It can be seen that the distributions match very well, although slight deviation is obtained in SNR=-4.3 dB.

Figs. 6 and 7 show the average BER and BLER, respectively, obtained using Blackbox and through simulations. The value of $p_{ideal}$ is taken directly from Blackbox #1 and $\text{BLER}_{ideal} = 1 - (1 - p_{ideal})^{2}$. We can see that in cases 1 and 2, the curves exactly match. For cases 3 and 4, it can be observed that the result provided by the Blackbox is optimistic compared to the simulations. Possible approaches to improve the accuracy are to improve the JD performance, improve the accuracy of the theoretical results used in Blackbox #1 and/or to find better matching error distributions. However, this investigation to improve the accuracy of the model in
Cases 3 and 4 is left as a future study.

V. APPLICATION

We have integrated the Blackbox into the popular ns-3 simulator and carried out a simulation-based performance evaluation for a representative scenario. This section explains the implementation design for the ns-3 enhancements and presents the performance results.

A. Integration of the Blackbox into Network Simulator ns-3

ns-3 is a free and open-source discrete-event simulator, which is well-accepted in the scientific community and widely used for performance evaluation of data networks and communication systems [18]. The simulator has an object-oriented structure and includes various modules for Wi-Fi, cellular and other system, which can be modified and extended. ns-3 is a discrete-event simulator, which models the operation of the system as a discrete sequence of events in time. Key components of a network based on ns-3 models are nodes, applications, channels and network devices, the latter model the link-level including PHY and MAC. For scalability reasons and to enable simulations at low computational costs, ns-3 implements PHY abstractions to predict the link layer performance. A MI-link quality model has already been integrated into the ns-3 distribution [19]. However, this model does not consider relaying. Therefore, we have implemented the error rate model for lossy forwarding and joint decoding and integrated it into ns-3.

For our studies we have used the ns-3 wifi module, which simulates the IEEE 802.11 PHY and MAC. The integration of the PHY abstraction takes place inside the WifiPhy and the yans classes, more specifically in the send and receive functions. We make use of packet tags, a ns-3 feature that allows appending additional information to a packet and thereby to enhance existing protocols by functions for lossy forwarding and joint decoding.

B. Simulation Scenario and Results

The network under consideration consists of a single source, multiple (two or three) relays and a single destination. We study the effect of joint decoding and quantify the advantage of using more relays with respect to the reliability. The nodes execute a WiFi-like PHY and MAC (IEEE 802.11a) extended by lossy forwarding and joint decoding. For multi-path packet routing we apply an enhanced version of contention-based geographical forwarding (CBGF) [20]. The channel is modeled by a log-distance path loss with a channel exponent of 3 and Nakagami distribution with shape factor 1, which corresponds to a Rayleigh fading channel. For details of the scenario and models we refer to [20].

We measure the packet success ratio (PSR) – the ratio of successfully received packets at the destination and the sent packets from the source – the number of packet copies (decoding attempts) required for error-free decoding – over the transmit power in dBm, whereas all nodes have the same transmit power. In LF+JD-On packets with errors are forwarded at the relays (lossy forwarding – LF) and combined at the destination (joint decoding - JD) whereas in LF+JD-Off the relays forward only those packets that are correct and at the destination joint decoding is disabled, thus the destination needs to wait until an error-free packet arrives.

To illustrate the functioning of the Blackbox, we consider the PSR with respect to the number of decoding attempts, which corresponds to the number of received packet copies, whereas the summation over the per-packet-copy PSR yields the overall PSR (Fig. 8 top Accumulated). The PSR grows for the two-copies and three-copies cases (Fig. 8 center and bottom) to a maximum and decreases for higher transmit power values. The reason for the decay is that with a high transmit power the destination is able to successfully decode the first arriving packet copy (see Fig. 8 top). For three packet copies, at approximately 25 dBm the two curves intersect (Fig. 8, bottom) because with LF+JD-On the Blackbox was already able to decode the packet correctly with 2 packet copies and does not require additional packet copies, whereas LF+JD-Off need to wait for the third error-free packet copy. If we regard the blue curve in Fig. 8 as the baseline then the difference between the red and blue curves gives the gain achieved by LF and JD.

2 The implementation of the Blackbox for ns-3 is available as open source at https://mns.infn.et.tu-dresden.de/Research/Projects/Pages/RESCUE.aspx.
accumulated PSR (sum of all packet copy curves). The error-bars refer to two standard deviations.

Fig. 8. Packet success ratio (PSR) over transmit power [dBm] for the number of decoding attempts (1, 2 or 3 packet copies). The top figures also show the accumulated PSR (sum of all packet copy curves). The error-bars refer to two standard deviations.

VI. CONCLUSION

We introduced a model that is capable of predicting the BLER in a wireless communication system with distributed turbo coding, where the destination receives multiple copies of potentially corrupted packets and reconstructs the source information. We have verified the accuracy of the model for the AWGN channel with several modulation and coding schemes. Furthermore, we have demonstrated the application of the error rate model in a protocol simulator and quantified the gains of distributed source coding approach in a state-of-the-art ad hoc routing protocol. The proposed model represents a PHY abstraction that is capable to accurately predict the link-level performance at low computational complexity and can therefore be utilized for the design and performance evaluation of algorithms and protocols that exploit distributed turbo coding as well as for system-level simulation.

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REFERENCES


Deliverables of the project “Links-on-the-fly Technology for Robust, Efficient and Smart Communication in Unpredictable Environment” listed as references can be found at http://www.ict-rescue.eu/rescue-deliverables.