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Multi-Cell Linear Precoding Design for Throughput Optimization With Imperfect CSI and Outage

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***Abstract*—Cooperative multi-cell MIMO techniques are well known for their outstanding capability to mitigate inter-cell interference by allowing user data to be jointly processed by several interfering base stations for the improvement of the system performance. This paper proposes an iterative algorithm that designs the precoding matrix for cooperative multi-cell MIMO downlink communications. As the demand for higher throughput and higher system resilience is envisioned for future wireless communication systems, the proposed algorithm takes the potential outage into consideration when channel state information is only partially available at the transmitter side. The aim is to maximize the sum user throughput considering outage with subject to power limitation at each base station. The joint optimization of the precoding matrix and the assigned transmission rate is solved via a 2-step alternating algorithm. The performance of the proposed method is evaluated by Monte Carlo simulations and compared with existing methods. Simulation results show that our proposed algorithm achieves performance gain than other referenced methods over the entire compared SNR region. Performance of all methods are also compared and analyzed when inter-cluster interference is present.**

I. Introduction

The benefit of Multi-User Multi-Input-Multi-Output (MU-MIMO) systems in achieving high throughput in wireless system is widely recognized. It exploits spatial diversity by deploying multiple antennas at both the transmitter and receiver. With available Channel State Information (CSI), the Base Station (BS) transmits data simultaneously to multiple users over the same time-frequency resource in order to achieve a system throughput that increases linearly with the number of antennas. In such MU-MIMO systems, the capacity is further assured by implementing precoding techniques. Non-linear precoding, such as the dirty paper precoding method, can achieve Shannon's capacity in a MIMO downlink. However, in practice linear precoding is more favorable due to the its low complexity and computational burden.

Though MU-MIMO remains as the core technology in our current cellular networks, a significant improvement in the system capacity and reliability is intensively sought after for future wireless communication system. One way to achieve this goal is by increasing the number of antennas at the BS. Since the number of antennas being placed in a BS is always limited by hardware constraints, a form of cooperation has been developed that can further exploit spatial diversity. This approach is termed *Coordinated Multi-Point Communication* (CoMP). It has been the subject of intensive research study for several years. The basic idea is to allow for the sharing of antennas among BSs that belong to the same cooperative cluster. The sharing virtually increases the number of antennas that a single BS can deploy. In return, the system throughput grows further in a linear manner with the growing number of available antennas. Other than improving the system capacity, CoMP can also ensure a consistent service quality in LTE wireless broadband networks. By coordinating and combining signals from multi-cell and multiple antennas, the inter-cell interference is mitigated. This mitigation technique enables mobile users to enjoy a consistent performance and quality whether they are closed to the center of an LTE cell or at its outer edges. This paper studies the linear precoding design for a CoMP broadcast channel.

In the literature, many methods of designing the linear precoding matrix for a MU-MIMO broadcast channel exist. One category of methods is based on the assumption that perfect CSI is available at the transmitter, such as in [1], [2]. Another category of methods assumes that the CSI is impaired by an estimation error or a feedback delay [3], [4]. Authors in [5] addressed a maximization problem under a sum power constraint and a perfect CSI by introducing an additional zero forcing constraint. This is referred to as the weighted sum rate (WSR) problem. [6] presented a solution for the WSR maximization problem with a sum power constraint under imperfect CSI. [7] discovered that the WSR maximization problem can be solved by means of the weighted sum mean square error (MSE) minimization assuming that minimum MSE (MMSE) receive filters are applied at the user equipment (UEs). None of the above methods were developed for a CoMP broadcast channel. In our previously proposed method [8], we extended the algorithm of [7] by using the approximation of [6]. This solution took into account that the CSI error variance can be different for each BS-UE link. It is designed for a multi-cell setup (CoMP broadcast channel) by consistently scaling the precoding matrix in order to satisfy the power constraint for each BSs. However, in the aforementioned method, outage was not considered.

Outage is an indication of the network reliability. When the actual channel is not able to support the assigned transmission rate, the transmitted data block are not guaranteed to be decoded completely. In this case, outage occurs and the transmission is in error. As reliability is becoming more and more crucial in the future wireless communication network, we are motivated to design a system that maximizes the reliably-transmitted throughput. From this aspect, we extend our previously proposed algorithm into considering outage.

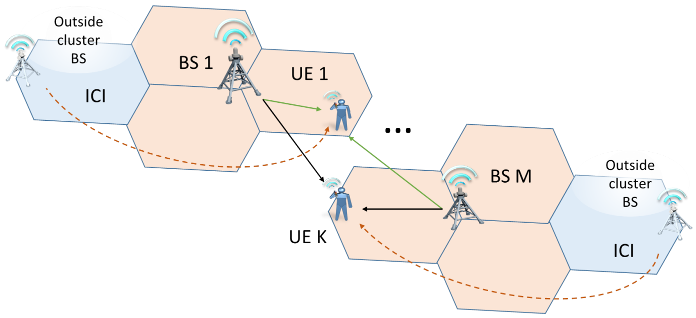

Fig. 1. A cellular system structure for cooperative multi-cell transmission in the downlink

We aim at optimizing the sum user throughput in a CoMP broadcast channel, considering that the transmission can be in outage with transmit power constraint at each BS. For this purpose, the objective function targets at a joint optimization of the precoding matrix and the assigned transmission rate. This complex problem is solved by means of a two-step multi-variate alternating algorithm. The lack of a closed form expression for the outage probability is overcome by making use of the Markov inequality extension [9] (Concentration Inequality) to translate the problem into a WSR maximization problem with adaptive weighting factors.

The remainder of this paper is organized as follows: The system model and problem formulation will be introduced in Sec. II and III, respectively. The optimization algorithm will be presented in Sec. IV. Numerical examples will be provided in Sec. V and finally, our conclusions will be drawn in Sec. VI.

Notation: Conjugate, transposition and conjugate transposition is denoted with $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$, respectively. The trace of a matrix is written as tr$(\cdot)$, $\det(\cdot)$ denotes determinant. The operator dg$(\cdot)$ replaces each non-diagonal element of a matrix with zero, while diag$(\cdot)$ places the elements of a vector on the diagonal of a matrix. Similarly, blkdiag$(\cdot)$ puts matrices on the diagonal of a block diagonal matrix. Expectation is denoted with $\mathbb{E}\{\cdot\}$ and probability by $\mathbb{P}\{\cdot\}$. $\mathbb{C}$ donates the set of complex numbers and $\mathcal{N}_{\mathbb{C}}(\mathbf{m}, \mathbf{C})$ refers to a multi-variate complex normal distribution with mean vector $\mathbf{m}$ and covariance matrix $\mathbf{C}$.

II. System Model

We consider a cooperative network with multiple wireless cells that form a cluster. Figure 1 illustrates a cellular system structure for cooperative multi-cell data transmission in the downlink.

In this figure, M BSs jointly serve K UEs on the same radio resource. All BSs within the cluster exchange user data and CSI over a backhaul network. All BSs within the cluster are assumed to be distributed with equal spacing in a single dimension.

BSs outside the cooperative cluster are assumed to be located symmetrically with respect to the cluster. The interference from outer cluster BSs receiver at the K UEs within the cluster is referred to as Inter-Cluster Interference (ICI) and is assumed to be Gaussian distributed [10]. The variance of ICI received at UE k is given as

$$\sigma^2_{\mathrm{I},k} = \eta\gamma\beta \sum_{i=1}^{M_{OC}/2} \left[(id_I + d_{k,1}^{-\alpha}) + (id_I + d_{k,M})^{-\alpha}\right], \quad (1)$$

where η indicates the isolation of a cluster, which is obtained in real 3-D dimensional setups by, e.g., antenna tilting. γ denotes the maximum transmit power at one BS and d_I indicates the inter-site distance, $d_{k,m}$ is the distance between k-th user to m-th BS, M_{OC} denotes the number of BSs that are outside the cooperative cluster, finally, β is the adjusting coefficient to account for shadow fading effects.

As shown in Figure 2, each BS m is equipped with B_m antennas, where $m = 1, \cdots, M$, and each UE k is equipped with U_k antennas, where $k = 1, \cdots, K$. Let $U = \sum_{k=1}^{K} U_k$ denotes the total number of antennas at the UE side, and $B = \sum_{m=1}^{M} B_m$ denotes the total number of antennas at the BS side. The data intended for user k is denoted as $\mathbf{d}_k$. Data to all users is concatenated and input to each base station as $\mathbf{d} = [\mathbf{d}_1^T, \cdots, \mathbf{d}_K^T]^T$. All elements of $\mathbf{d}$ are assumed to be independent and identically distributed with zero mean and unit variance, $\mathbf{d} \sim \mathcal{N}_C(\mathbf{0}, \mathbf{I})$. Before transmission, $\mathbf{d}$ is multiplied with the precoding matrix $\mathbf{B}$, which is defined as

$$\mathbf{B} = [\bar{\mathbf{B}}_1, \cdots, \bar{\mathbf{B}}_K] = [\mathbf{B}_1^T, \cdots, \mathbf{B}_M^T]^T, \quad (2)$$

where $\mathbf{B}_m \in \mathbb{C}^{[B_m \times U]}$ denotes the portion of the precoding matrix that is applied at the m-th BS, while $\bar{\mathbf{B}}_k \in \mathbb{C}^{[B \times U_k]}$ denotes the portion of the precoding matrix that is applied to precode the data intended for the k-th UE. In order to meet the per-BS transmit power constraints, the precoding matrix needs to satisfy the condition tr$(\mathbf{B}_m \mathbf{B}_m^H) \leq \gamma_m$, $\forall\ m$.

The precoded data is then coherently transmitted from all M BSs to all K UEs over a multi-cell broadcast channel which is denoted by $\mathbf{H} \in \mathbb{C}^{[U \times B]}$. Each element in the matrix $\mathbf{H}$ corresponds to the channel state of the link between a particular transmit and receive antenna pair. In this work, block-static fading is assumed where the channel state during a transmission block remains constant and subsequent blocks are not correlated. According to Rayleigh fading, each link between BS m and UE k is complex Gaussian distributed with zero mean and variance $\lambda_{k,m}$. The channel matrix can be expressed as:

$$\mathbf{H} = [\mathbf{H}_1^T, \cdots, \mathbf{H}_K^T]^T, \quad (3)$$

where the sub-matrix $\mathbf{H}_k \in \mathbb{C}^{[U_k \times B]}$ corresponds to the channel links between all BS antennas and the antennas of UE k.

At each UE, the received signals are equalized by a linear receive filter $\mathbf{U}_k$. Let $\mathbf{y}$ denote the received symbols before filtering. It reads

$$\mathbf{y} = [\mathbf{y}_1^T, ..., \mathbf{y}_K^T]^T = \mathbf{HBd} + \mathbf{n}, \quad (4)$$

where $\mathbf{n} = [\mathbf{n}_1^T, \cdots, \mathbf{n}_K^T]^T$ is the overall noise vector. Let $\hat{\mathbf{d}}$ denote the processed symbols after filtering. It is obtained by

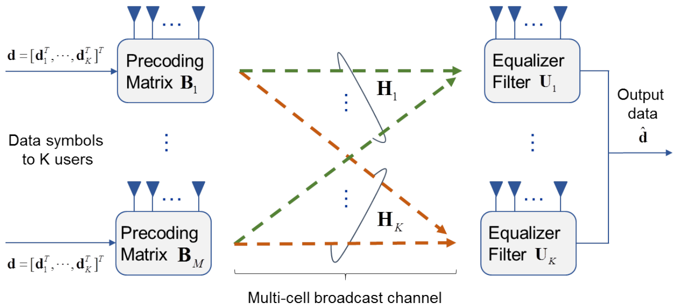

Fig. 2. Model of a cooperative multi-cell MIMO system

stacking the equalized data symbols of all K UEs into a single vector, as

$$\hat{\mathbf{d}} = \mathbf{U}(\mathbf{HBd}+\mathbf{n}) = \mathbf{Uy}, \tag{5}$$

where the matrix $\mathbf{U} = \text{blkdiag}(\mathbf{U}_1, \cdots, \mathbf{U}_K)$ represents the equalization function.

With this system model, we will move on to the problem formulation in the next section.

III. Problem Formulation

In practice, CSI is only imperfectly available at the BS side. It is often impaired by channel errors or delayed feedback. Assuming that CSI is obtained via minimum MSE (MMSE) estimation, imperfections in the CSI can be modeled by

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}\,, \tag{6}$$

where $\hat{\mathbf{H}}$ denotes the estimated CSI matrix, which is uncorrelated with the Gaussian error matrix $\mathbf{E} = [\mathbf{E}_1^T, \cdots, \mathbf{E}_K^T]^T$ with $\mathbf{E}_k = [\mathbf{E}_{k,1}, \cdots, \mathbf{E}_{k,M}]$ and $\mathbf{E}_{k,m} \sim \mathcal{N}_\mathbb{C}(\mathbf{0}, \epsilon_{k,m}\mathbf{I}))$.

Imperfections in the CSI cause inaccuracies in the calculation of precoding matrix. As a consequence, the precoding matrix does not perfectly fit to the actual channel matrix. Furthermore, rate adaptation suffers from impaired CSI, since it leads to mismatches between the assigned transmission rate and the actual data rate that a channel can support. When the assigned transmission rate is less than the actual data rate, outage occurs. The probability of outage is expressed as

$$p_{\text{out},k} = \Pr(R_k < \bar{R}_k), \tag{7}$$

where R_k is the actual data rate which is supported by the channel for user k, and $\bar{R}_k$ is the assigned transmission rate for this user.

Inspired by the latest motivation for 5G cellular networks, which has higher demand on data throughput and network reliability than the current 4G networks [11], we are interested in investigating the design of the precoding matrix when outage is taken into consideration.

By combining all user specific transmission rates into a single vector $\bar{\mathbf{r}} = [\bar{R}_1, \cdots, \bar{R}_K]^T$, the following formulation of the problem is reached:

$$\begin{aligned}[\mathbf{B}^\star, \bar{\mathbf{r}}^\star] = \quad & \arg\max_{\mathbf{B}, \bar{\mathbf{r}}_k} \quad \textstyle\sum_{k=1}^K (1-p_{\text{out},k})\bar{R}_k\,,\\ & \text{s.t.} \quad \text{tr}(\mathbf{B}_m\mathbf{B}_m^H) \le \gamma_m \quad \forall m,\end{aligned} \tag{8}$$

where $\mathbf{B}^\star$ and $\bar{\mathbf{r}}^\star = [\bar{R}_1^\star, \cdots, \bar{R}_K^\star]^T$ denote the optimized precoding matrix and the optimized transmission rate vector consisting of the optimized rate for UE k. This objective function aims at maximizing the sum user throughput with per-BS transmit power constraints when CSI is imperfect and the outage is considered. It is a joint optimization of the precoding matrix and the assigned transmission rate.

Solving this problem is not a trivial task. The reason being, first of all, the closed form expression of the outage probability is not known. Secondly, outage probability is a function of the precoding matrix and the assigned transmission rate. This optimization problem has optimization variables that are inter-coupled with one another. In this work we present a solution which translates the joint optimization into two optimization problems and solve it via an alternating algorithm.

Introducing an approximation, a suboptimal solution is found for the precoder optimization problem.

IV. Optimization Algorithm

A 2-step alternating algorithm is proposed to solve the above objective function. The basic idea is to iterate between the optimal assigned transmission rate and the precoding matrix, until the overall metric converges to a fixed point. There are 2 steps in each iteration. In the first step, we obtain the optimal assigned transmission rate $\bar{R}_k^\star$ by fixing the precoding matrix $\mathbf{B}$. In the second step, we compute the precoding matrix $\mathbf{B}$ using the optimal transmission rate $\bar{R}_k^\star$. After both variables have been updated, a new iteration is executed. It is known that with alternating optimization a local optimum can be obtained, while global optimality cannot be guaranteed unless the function subject to optimization is smooth and convex [12].

1) Step 1: In order to obtain the optimal assigned transmission rate, we have to fix a precoding matrix. To obtain the initial precoding matrix $\mathbf{B}$, we assume the outage probability to be zero and CSI to be imperfect. Hence, the objective function for calculating the initial precoding matrix is

$$\begin{aligned}\mathbf{B}^\star = \quad & \arg\max_{\bar{\mathbf{B}}} \quad \textstyle\sum_{k=1}^K R_k\,,\\ & \text{s.t.} \quad \text{tr}(\mathbf{B}_m\mathbf{B}_m^H) < \gamma_m \quad \forall m.\end{aligned} \tag{9}$$

Solution for solving this objective function under imperfect CSI was presented in [8]. With this initial precoding matrix, we then optimize the transmission rate for a given channel estimate. The objective function is written as

$$\begin{aligned}\bar{\mathbf{r}}^\star = \quad & \arg\max_{\bar{\mathbf{r}}} \quad \textstyle\sum_{k=1}^K (1-\mathbb{P}\{R_k(\mathbf{B}) < \bar{R}_k\})\bar{R}_k\,,\\ & \text{s.t.} \quad \bar{R}_k \ge 0 \quad \forall k.\end{aligned} \tag{10}$$

The transmission rate $\bar{R}_k$ in (10) is obtained by substituting the imperfect channel matrix in (6) in the following equation,

$$R_k = \log\det(\mathbf{I} + \mathbf{H}_k\bar{\mathbf{B}}_k\bar{\mathbf{B}}_k^H\mathbf{H}_k^H\mathbf{C}_k^{-1})\,, \tag{11}$$

where $\mathbf{H}_k\bar{\mathbf{B}}_k\bar{\mathbf{B}}_k^H\mathbf{H}_k^H$ refers to the meaningful portion of the signal received at user k, while $\mathbf{C}_k = \sum_{l\neq k}\mathbf{H}_k\bar{\mathbf{B}}_l\bar{\mathbf{B}}_l^H\mathbf{H}_k^H + \sigma_{n,k}^2\mathbf{I}$ indicates noise and interference to this user. Outage probability $\mathbb{P}\{R_k(\mathbf{B}) < \bar{R}_k\}$ is generated with respect to error $\mathbf{E}$ using Monte-Carlo simulations for each transmission rate $\bar{R}_k$. $\bar{R}_k^\star$ is obtained when $\bar{R}_k$ gives the maximum overall metric in (10).

2) Step 2: With $\bar{R}_k^\star$ obtained in the first step, the joint optimization in (8) is reduced to

$$\begin{aligned} \mathbf{B}^\star = \quad & \arg\max_{\mathbf{B}} \quad \textstyle\sum_{k=1}^K -\mathbb{P}\{R_k(\mathbf{B}) < \bar{R}_k^\star\}\bar{R}_k^\star \, . \\ & \text{s.t.} \quad \text{tr}(\mathbf{B}_m\mathbf{B}_m^H) < \gamma_m \quad \forall m \end{aligned} \tag{12}$$

Applying the concentration inequality [9] (an extension of the Markov inequality) gives the relation

$$\mathbb{P}\{x \geq a\} = \mathbb{P}\{f(x) \geq f(a)\} \leq \mathbb{E}\{f(x)\}/f(a)\,, \tag{13}$$

for a strictly increasing and non-negative function $f(x)$, and the following transformation can be applied to (12)

$$\mathbb{P}\{R_k < \bar{R}_k^\star\} = 1 - \mathbb{P}\{R_k \geq \bar{R}_k^\star\} \geq 1 - \mathbb{E}\{f(R_k)\}/f(\bar{R}_k^\star)\,. \tag{14}$$

Let $f(x) = a + bx$, with $a, b \in \mathbb{R}_0^+$, the objective function is transformed into

$$\begin{aligned} \mathbf{B}^\star = \quad & \arg\max_{\mathbf{B}} \quad \Sigma_{k=1}^K \nu_k \mathbb{E}\{R_k\}\,, \\ & \text{s.t.} \quad \text{tr}(\mathbf{B}_m\mathbf{B}_m^H) < \gamma_m \quad \forall m \end{aligned} \tag{15}$$

with the user specific coefficients

$$\nu_k = b \cdot \bar{R}_k^\star/(a + b \cdot \bar{R}_k^\star). \tag{16}$$

where a and b are arbitrary constants.

To solve (15), first, the achievable rate expression in (11) is rewritten as

$$R_k = -\log\det(\mathbf{M}_k)\,, \tag{17}$$

with the MMSE covariance matrix

$$\mathbf{M}_k = (\mathbf{I} + \bar{\mathbf{B}}_k^H \mathbf{H}_k^H \mathbf{C}_k^{-1} \mathbf{H}_k \bar{\mathbf{B}}_k)^{-1}\,. \tag{18}$$

This relation is proved in [7]. Based on (17), and applying Jensen's inequality [6], the expected achievable rate in (15) can be lower bounded by

$$\mathbb{E}\{R_k\} = \mathbb{E}\{-\log\det(\mathbf{M}_k)\} \geq -\log\det(\mathbb{E}\{\mathbf{M}_k\}) = \breve{R}_k, \tag{19}$$

Hence, the optimization problem in (15) is transformed into a max-min problem.

$$\begin{aligned} \mathbf{B}^\star = \quad & \arg\max_{\bar{\mathbf{B}}} \quad \textstyle\sum_{k=1}^K \breve{R}_k\,, \\ & \text{s.t.} \quad \text{tr}(\mathbf{B}_m\mathbf{B}_m^H) < \gamma_m \quad \forall m. \end{aligned} \tag{20}$$

The problem in (20) is of the form as the WSR maximization problem. A solution of this optimization problem can be found by alternatingly optimizing the MMSE filter matrix $\mathbf{U}$ and the precoding matrix $\mathbf{B}$ in an iterative manner [8].

Following the derivation in [6], with the imperfect CSI in (6),

$$\begin{aligned} \bar{\mathbf{M}}_k \quad & = E_{\mathbf{E}_k}\{\mathbf{M}_k\} = E_{\mathbf{E}_k}\{(\mathbf{d}_k - \mathbf{U}_k\mathbf{y}_k)(\mathbf{d}_k - \mathbf{U}_k\mathbf{y}_k)^H\} \\ & = \mathbf{I} + \mathbf{U}_k(\hat{\mathbf{H}}_k\mathbf{B}\mathbf{B}^H\hat{\mathbf{H}}_k^H + \sigma_n^2\mathbf{I})\mathbf{U}_k^H + \zeta_k\mathbf{U}_k\mathbf{U}_k^H \\ & -\mathbf{U}_k\hat{\mathbf{H}}_k\bar{\mathbf{B}}_k - \bar{\mathbf{B}}_k^H\hat{\mathbf{H}}_k^H\mathbf{U}_k^H\,. \end{aligned} \tag{21}$$

where $\zeta_k = [\epsilon_{k,1}\mathbf{1}_{1\times B_1}, \cdots, \epsilon_{k,M}\mathbf{1}_{1\times B_M}] \cdot \text{diag}(\mathbf{B}\mathbf{B}^H)$.

Algorithm 1 Proposed 2-step alternating algorithm

Input: $\hat{\mathbf{H}}$, σ_n^2, ρ_m, ϵ
Output:$\mathbf{B}$, $\bar{R}_k$
Iteration index $i \leftarrow 0$
Init: $\mathbf{B}^i$=$\mathbf{B}^{\text{init}}$ according to [8], $\nu_k = 1$, $\forall k$
repeat
  Update $i = i + 1$
  (a) Compute the optimized transmission rate $\bar{R}_k^\star$, $\forall k$ by solving (10)
  (b) Update weights ν_k, $\forall k$ with (16)
  (c) Calculate $\mathbf{B}^i$ with updated ν_k
until convergence

With (21), the sum MSE of each UE can be expressed as $\bar{\epsilon}_k = tr(\bar{\mathbf{M}}_k)$. By setting the derivative of $\bar{\epsilon}_k$ with respect to $\mathbf{U}_k$ to be zero, we obtain the expression of MMSE filter at the receiver of user k as follows:

$$\mathbf{U}_k^{\text{MMSE}} = \mathbf{B}_k^H\hat{\mathbf{H}}_k^H(\hat{\mathbf{H}}_k\mathbf{B}\mathbf{B}^H\hat{\mathbf{H}}_k^H + (\sigma_{n,k}^2 + \zeta_k)\mathbf{I})^{-1}. \tag{22}$$

The MMSE matrix $\bar{\mathbf{M}}_k^{\text{MMSE}}$ can then be obtained by substituting (22) into (21). It was shown in [7] and [8] that the WSR maximization problem of the form in (20) is equivalent to a Weighted-sum MMSE (WMMSE) minimization problem as follows if the weighting matrices $\mathbf{W}_k = \nu_k(\bar{\mathbf{M}}_k^{MMSE})^{-1}$

$$\begin{aligned} \mathbf{B}^\star = \quad & \arg\min_{\mathbf{B}} \quad \textstyle\sum_{k=1}^K \text{tr}(\mathbf{W}_k\bar{\mathbf{M}}_k) \\ & \text{s.t.} \quad \text{tr}(\mathbf{B}_m\mathbf{B}_m^H) < \gamma_m \quad \forall m, \end{aligned} \tag{23}$$

where

$$\bar{\mathbf{M}}_k^{\text{MMSE}} = (\mathbf{I} + \mathbf{B}_k^H\hat{\mathbf{H}}_k^H\bar{\mathbf{C}}_k^{-1}\hat{\mathbf{H}}_k\mathbf{B}_k)^{-1}, \tag{24}$$

$$\bar{\mathbf{C}}_k = (\sigma_n^2 + \zeta_k)\mathbf{I} + \sum_{l=1, l\neq k}^{K} \hat{\mathbf{H}}_k\mathbf{B}_l\mathbf{B}_l^H\hat{\mathbf{H}}_k^H. \tag{25}$$

A solution for the optimization problem in (23) is given by the transmit Wiener filter approach and is obtained by the following expression

$$\mathbf{B} = \tau\left(\hat{\mathbf{H}}^H\mathbf{U}^H\mathbf{W}\mathbf{U}\hat{\mathbf{H}} + \text{dg}(\mathbf{D}) + \frac{\sigma_k^2\text{tr}(\mathbf{W}\mathbf{U}\mathbf{U}^H)}{\gamma}\mathbf{I}\right)^{-1}\hat{\mathbf{H}}^H\mathbf{U}^H\mathbf{W}, \tag{26}$$

where $\mathbf{D} = \mathbf{G}^T\text{diag}(\mathbf{U}^H\mathbf{W}\mathbf{U})\mathbf{1}_{1\times U}$ is the regularization matrix, which accounts for CSI imperfections. Furthermore, $\mathbf{W} = \text{blkdiag}(\mathbf{W}_1, \cdots, \mathbf{W}_K)$ denotes the overall weighting matrix and the extended error variance matrix $\mathbf{G} = [\mathbf{G}_1^T, \cdots, \mathbf{G}_K^T]^T$ with $\mathbf{G}_k = [\mathbf{G}_{k,1}, \cdots, \mathbf{G}_{k,M}]$, which consists of the link-wise error variances $\mathbf{G}_{k,m} = \epsilon_{k,m} \cdot \mathbf{1}_{U_k \times B_m}, \forall k, m$. In order to satisfy the per-BS transmit power constraints, the scaling factor

$$\tau = \max_m \sqrt{\gamma_m/tr(\hat{\mathbf{B}}_m\hat{\mathbf{B}}_m^H)} \tag{27}$$

is applied.

The proposed algorithm is summarized in pseudo-code in **Algorithm 1**.

V. Numerical Examples

In this section the performance for the proposed algorithm will be evaluated and compared. We will first compare the results of our proposed algorithm with all reference methods when outage is considered. In the next step, the effect of inter-cluster interference will be included.

In all simulations, we assume the number of BSs is two, each with two transmit antennas; the number of UEs is two and each UE is equipped with two receive antennas. Radio propagation is modeled according to an urban macro-cell scenario as defined by 3GPP [13], where the inter-site distance d_I is 500 meters, $\beta = -144.5$ dB, and $\alpha = 3.5$. The two UEs are located at the cell edge.

According to the 1-D setup, introduced in Sec. II, the distance to the BSs is assumed to be $d_{k,m} = d_I/2, \forall\ k, m$. Without loss of generality, the variance of the CSI imperfection is assumed to be $\epsilon_{k,m} = 0.1 \cdot \lambda$, $\forall\ k, m$. Other values of $\epsilon_{k,m}$ are possible, but the basic behavior of the algorithm remains the same.

For all algorithms, Monte Carlo simulations with $10,000$ channel realizations were executed. For our proposed algorithm, 100 iterations for the outer loop and 100 iterations for the inner loop were computed. Outer loop refers to the alternating computation of precoding matrix and transmission rate, i.e., index i in **Algorithm 1**.

Inner loop corresponds to the calculation of the precoding matrix as given in [8]. "I-CSI" stands for imperfect CSI. "Outage" indicates that outage is considered and "ICI" means that inter-cluster interference is included.

Figure 3 compares the performances of our proposed algorithm and the two other reference methods, namely, the methods presented in [7] and [8]. The relationship between these three methods are such that [7] proposed solving the WSR problem using an iterative WMMSE algorithm, [8] extended this algorithm to imperfect CSI and our proposed method extends [8] considering outage. Figure 3 shows the throughput as the primary performance metric for the three aforementioned methods when the transmission is in outage. Because the two reference methods did not consider outage in their original respective design, they would yield poor results when data are transmitted at the rate obtained by assuming perfect CSI. To allow for a fair comparison, we assume that all three methods are aware of potential outage and data are transmitted at the optimized transmission rates $\bar{\mathbf{r}}^{\star}$. The solid lines denote the performances of three methods when no ICI is present, while the dotted lines refer to the performance of three methods when ICI is present. From the figure, we observe that, when no ICI is present, our proposed algorithm shows performance gains compared to [8] for the whole SNR regime, while the gain slightly increases with the SNR. At SNR = 30 dB, the gain compared to [8] is 0.5 bpcu and compared to [7] it is 2 bpcu.

The improvement of our proposed algorithm is due to the alternating optimization of precoding matrix and transmission rate, where each iteration increases the throughput. However, we observe that as the SNR value approaches to large value, the gain between our proposed method and [8] remains relatively constant. One possible reason would lie in the performance

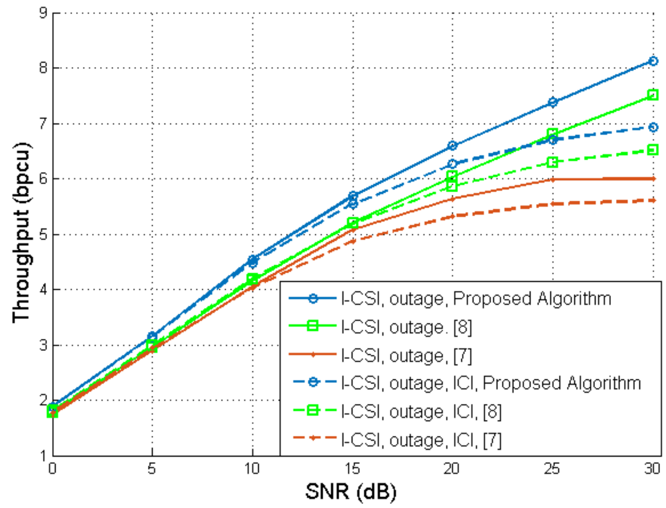

Fig. 3. Throughput in *bpcu* against SNR considering outage with/without inter-cluster interference for various algorithms

limit of the approximation. Loss in performance may incur depending on the strictness of the approximation, hence result in the saturation in performance when SNR is high. More results in this aspect will be presented in our future work.

When ICI is present, the performance of all methods are reduced at high SNR mainly, while the relations between the presented algorithms are similar to the case without ICI. At SNR = 30 dB our proposed algorithm obtains 6.9 bpcu, while [8] achieves 6.4 bpcu. These results are roughly 15% degraded compared to the no-ICI cases. From this simulation, we further observe that our proposed method, which works for CoMP, is efficient in minimizing the interference within the cooperative cluster. However, it is sensitive to interference from outside the cluster, especially at a high SNR regime.

VI. Conclusion

In this work, we presented the design of an algorithm for linear precoding in a cooperative MIMO downlink system, when channel state information (CSI) is only partially available at the BS side and transmissions can be in outage. The algorithm targets at the maximization of the sum user throughput by jointly optimizing the precoding matrix and the transmission rate using a 2-step alternating algorithm. We evaluated our proposed algorithm by means of Monte Carlo simulations, and compared the proposed algorithm with state-of-the-art solutions. It was shown that a performance gain in terms throughput can be achieved.

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